Series 3

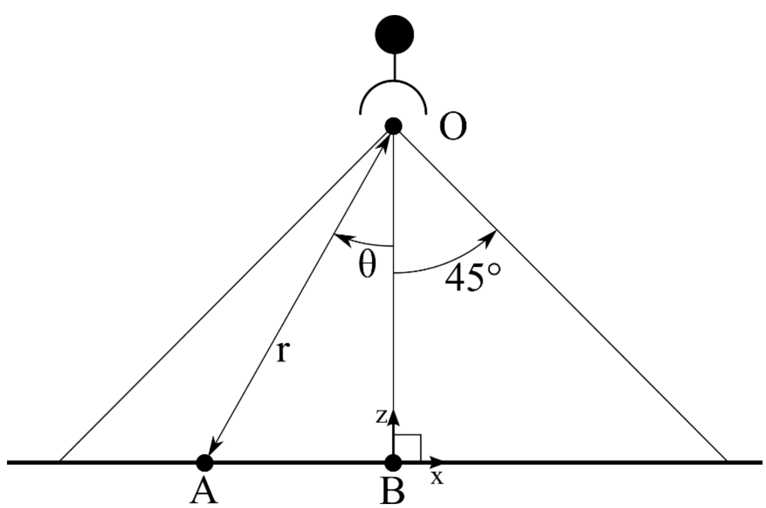
Exercise 1

An airplane is flying at an altitude of 2km above the ground, and has an antenna mounted under its fuselage. This antenna should transmit a power density which should be constant n the ground within a cone of which summit is the antenna. The revolution axis of the cone is the vertical line between the antenna and the ground, the airplane is the summit of the cone and the aperture angel of the cone from this axis is of 45°. The antenna should not transmit outside of the cone, and the ground is perfectly flat.

1) Find the expression of the radiation pattern as a function of θ measured form the cone axis.
2) Compute the directivity of the antenna ?

Solution :

This is a very real problem. We do know that the power density decreases in $1/r^2$. As the trajectory OB is shorter than OA, there will a priori be more power density in b than in A. This unbalance needs to be compensated by the radiation pattern.



We know that the power density transmitted by the antenna has the following form: :

$$p(r,\theta,\varphi) = \frac{C}{r^2} D_p(\theta,\varphi) \left[W / m^2 \right]$$

To compute $D_p(\theta,\varphi)$ such as p=cte, we need to express r as a function of θ:

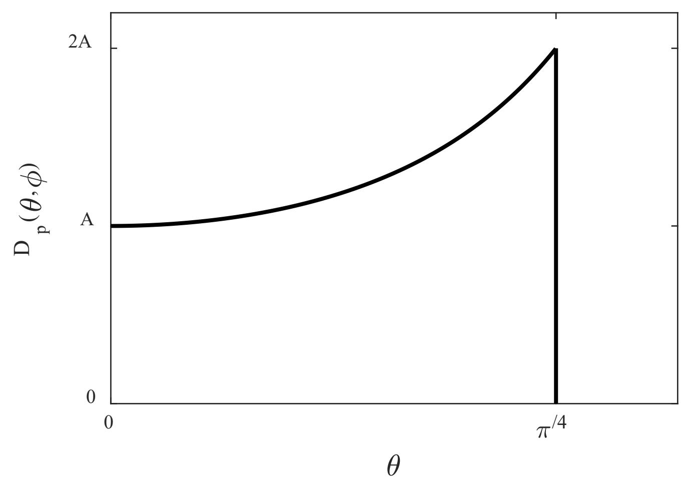
$$r = \frac{h}{\cos\theta}$$

From which

$$p = \frac{\cos^2\theta}{h^2} D_p(\theta,\varphi) = cte \quad \rightarrow$$

$$D_p(\theta,\varphi) = \frac{A}{\cos^2\theta} \quad \theta < \frac{\pi}{4}$$

Where A is a constant independent of θ and φ. The final radiation pattern is thus given by:



Which can be normalized by dividing by 2A. The directivity is given by:

$$D = 4\pi r^2 \frac{p(r,\theta,\varphi)}{P_{rad}} = 4\pi \frac{\frac{1}{\cos^2\theta}}{\int_0^{2\pi} d\varphi \int_0^{\pi/4} d\theta \frac{1}{\cos^2\theta}\sin\theta} = \frac{2}{\sqrt{2}-1}\frac{1}{\cos^2\theta}$$

Which maximal value is

$$D_{\max} = D(\theta = \pi/4) = \frac{4}{\sqrt{2}-1} = 9.66 = 9.85dB$$

Exercise 2

At a certain frequency, the transmission between two antennas gives a ratio between received power and transmitted power of -60dB. One of the antenna is a parabolic reflector, the other a very short dipole of which the directivity does not depend on the frequency over a large frequency band. We now double the frequency. What is now the new ration between received power and transmitted power ?

Solution:

The fundamental equation for this problem is Frii's formula:

$$\frac{P_{av-r}}{P_e} = \left(\frac{\lambda}{4\pi r}\right)^2 D_1 D_2$$

$$\frac{A_e}{D} = \frac{\lambda^2}{4\pi}$$

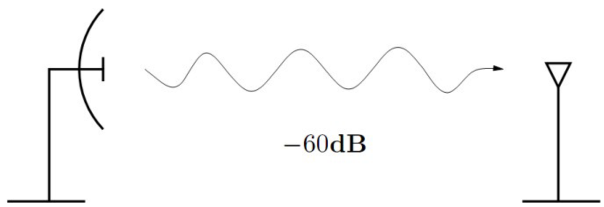


Figure 1.

$$\frac{P_{av-r}}{P_e} = \left(\frac{\lambda}{4\pi r}\right)^2 D_1 D_2 \qquad (1)$$

where D_1, D_2 are the directivity of the two antennas and r is the distance between them, λ being the wavelength

The dipole directivity is given by $D_1 = \frac{3}{2} sin^2\theta$ *(can be considered frequency independent).*

For the parabola, we need to find D_2 as a function of λ. The link between the directivity D_2 of an antenna and its effective area A_{e2} is given by. We may suppose A_{e2} is frequency independent for a parabola.

$$D_2 = \frac{4\pi}{\lambda^2} A_{e2} \qquad (2)$$

Using (1) and (2), we obtain

$$\frac{P_{av-r}}{P_e} = \left(\frac{\lambda}{4\pi r}\right)^2 D_1 \frac{4\pi}{\lambda^2} A_{e2} = \frac{D_1 A_{e2}}{4\pi r^2} = const. (independent\ of\ the\ freqeuncy)$$

In this particular case, we see that the power ration is frequency independent. This stems from the (realistic) assumptions that the effective area of the parabola and the directivity of tee dipole are frequency independent over a large band.

Exercise 3

Three lossless antennas A, B and C operating at 10GHz have unknown directivities, that we want to determine. To this aim, we realize a transmission system with a measurement distance of 1 m between the antennas, and a transmission power of 1W. We supposed the antennas are matched to the transmitter or the receiver, respectively. We test different antenna combinations on the transmitter and the receiver, and measure the received powers (always orienting the antenna such as to maximize this received power):

Pr = 6 mW for a tranmission between *A* and *B*,

Pr = 10 mW for a tranmission between *A* and *C*,

Pr = 1 mW for a tranmission between *B* and *C*.

What is the maximal directivity oof each antenna ?

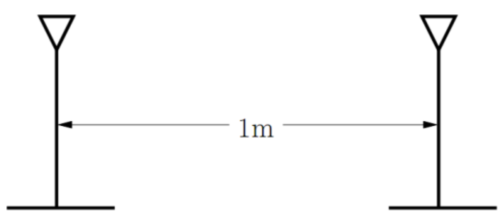


Figure 2

Solution:

We use Frii's formula

$$\frac{P_{av-r}}{P_e} = \left(\frac{\lambda}{4\pi r}\right)^2 D_1 D_2 = k D_1 D_2, \quad k = \left(\frac{\lambda}{4\pi r}\right)^2$$

In the case of an optimal orientation, we obtain the following transmission coefficients:

$$\eta_{AB} = \frac{P_{av-r}^{A}}{P_e^B} = kD_A D_B = 6x10^{-3}$$

$$\eta_{AC} = \frac{P_{av-r}^{A}}{P_e^C} = kD_A D_C = 10x10^{-3}$$

$$\eta_{BC} = \frac{P_{av-r}^{B}}{P_e^C} = kD_B D_C = 1x10^{-3}$$

These three equations constitute a non linear system of three equations with three unknowns. The solution is given by :

$$D_A = \sqrt{\frac{\eta_{AB}\eta_{AC}}{k\eta_{BC}}} = 102.6$$

$$D_B = \sqrt{\frac{\eta_{AB}\eta_{BC}}{k\eta_{AC}}} = 10.26$$

$$D_C = \sqrt{\frac{\eta_{AC}\eta_{BC}}{k\eta_{AB}}} = 17.1$$